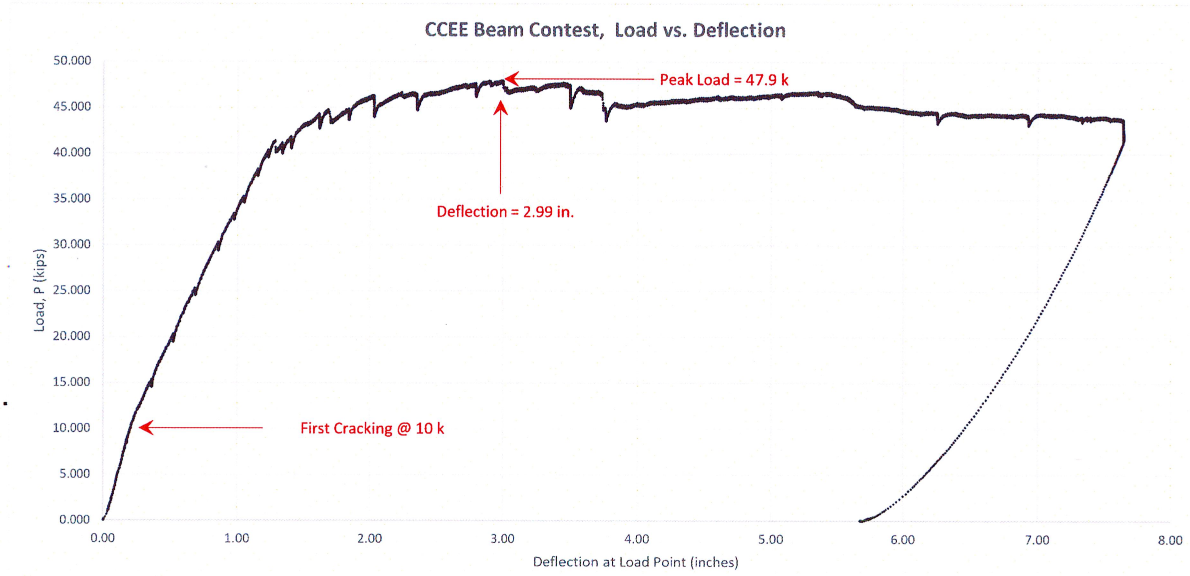
CCEE Beam Contest Results

This contest began as a lab project for our Spring 2016 CE334 class here at ISU. Volunteers from the class built the beam in late March and tested it on April 28 after it had cured for 28 days. The beam sustained a maximum load of 48 kips at a deflection of almost 3" (see Figure 1). The beam was very ductile, still supporting nearly 45 kips after undergoing over 7" of deflection. Cracking was first observed at the fixed support at around 10 kips. First yielding also appeared to occur near the fixity with the initiation of a plastic hinge at a load of about 40 kips. Thereafter, moments redistributed toward the mid-span of the beam that still maintained some reserve moment capacity. A second hinge formed at the point of loading as the beam softened, and loading reached its peak of 47.9 kips. After reaching its peak load, the beam continued to soften gradually as the plastic hinges underwent free rotation. The test was finally terminated when the loading ram began to lose uniform contact with the beam.



I want to thank the many practicing engineers, graduate students, and undergraduate students from outside the class who participated in predicting the load capacity of the beam. The contest apparently went viral on the web, attracting entries from such luminaries as Mickey Mouse, Taylor Swift, and the always creative, asdfjkl;. The winners, however, were required to be real people with e-mail addresses. Including students in CE334, over 150 predictions were submitted. The top three load capacity predictions are listed below in each of 3 categories: practicing engineers, graduate students, and undergraduate students. Congratulations to the winners – may the bragging rights carry you far!

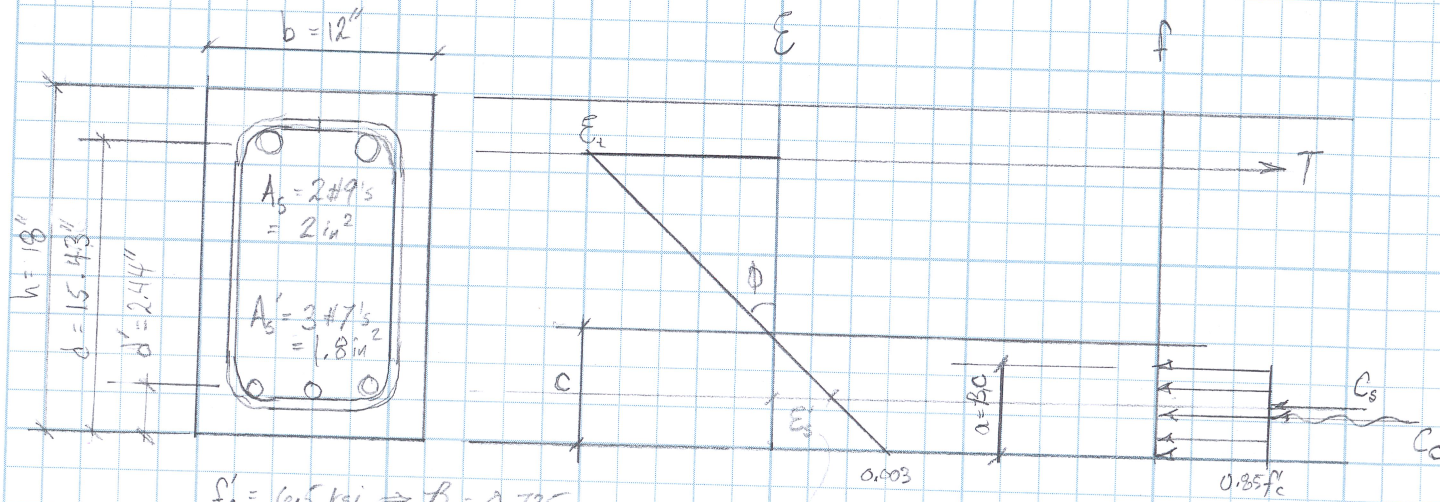
Category I: Practicing Engineers	**Category II: Grad Students**	**Category III: Undergrad Students**
1st – John Rhodes (43 kips)	1st - Seyedbabak Momenzadeh (45 kips)	1st – Molly Schmerbach (50 kips)
2nd – David Cockrum (41.3 kips)	2nd – Jonathan Martin (54 kips)	2nd – Matt Lyons (42.6 kips)
3rd – Jerad Hoffman (41 kips)	3rd – Dena Khatami (38 kips)	3rd – Joe Pape (39.6 kips)
		4th – Gage DeCook (56.8 kips)
		5th – Cody Harvey (37.8 k)
		6th – Kenton Johnston (36.3 k)

Interestingly, of the 12 people listed above, only 3 used any software (one used Enercalc, and the other two used SAP 2000). Many software packages were used by other participants including general FEM packages (ABAQUS, LS-DYNA), general linear-elastic frame analysis programs (SAP2000, STAAD, RISA), and specific R/C design programs (Enercalc, Consys). Roughly 50% of the entrants excluding CE334 students used some form of software while the rest relied on hand calculations or intuition. Generally, predictions using software tended to stray farther from the mark.

Finally, most predictions tended to be low. I suspect that this can be attributed to strain hardening of the reinforcing bars. The stress-strain plot shows that the bars began to strain harden almost immediately upon yielding. That fact coupled with the high ductility and large strains in the tension bars supplied significant additional load capacity.

Thank you all for playing. I hope the contest provided some entertainment. I know I've had fun with this!

Negative Moment Capacity @ Fixity:



$f'_c = 6.5\text{ ksi} \Rightarrow \beta_1 = 0.725$

$f_y = 69\text{ ksi} \Rightarrow \varepsilon_y = 69/29{,}000 = 0.0024$

$$\frac{\varepsilon_s'}{(c-2.44)} = \frac{0.003}{c} \Rightarrow \varepsilon_s' = \frac{(0.003)(c-2.44)}{c}$$

Assume A_s yields, A_s' does not

$$\Sigma F_x = 0 \Rightarrow C_c + C_s = T$$

$$(0.85f'_c)(\beta_1 c b - A_s') + A_s'\varepsilon_s' E_s = A_s f_y$$

$$(0.85)(6.5)[(0.725)(c)(12) - 1.8] + 1.8(29{,}000)\frac{(0.003)(c-2.44)}{c} = 2(69)$$

$$\Rightarrow \underline{c = 2.73''}$$

note: $\varepsilon_s' = 0.00032 < \varepsilon_y$ ✓

$\varepsilon_t = \dfrac{0.003(15.43-2.73)}{2.73} = 0.014 > \varepsilon_y$ ✓ (highly ductile)

$$\Rightarrow C_c = (0.85)(6.5)[(0.725)(2.73)(12) - 1.8] = 121.28^k\ (c)$$

$$C_s = 1.8(29{,}000)\frac{(0.003)(2.73-2.44)}{2.73} = 16.64^k\ (c)$$

$$T = 2(69) = 138^k\ (t)$$

check: $\Sigma = -0.09 \approx 0$ ✓

$$\Sigma M = 0 \Rightarrow M_n = C_c\left(d - \frac{a}{2}\right) + C_s(d - d')$$

$$= 121.28\left(15.43 - \frac{(0.725)(2.73)}{2}\right) + 16.64(15.43 - 2.44) = 1967.5\text{ in.-k}$$

$$= \underline{\underline{164\text{ ft.-kips}}}$$

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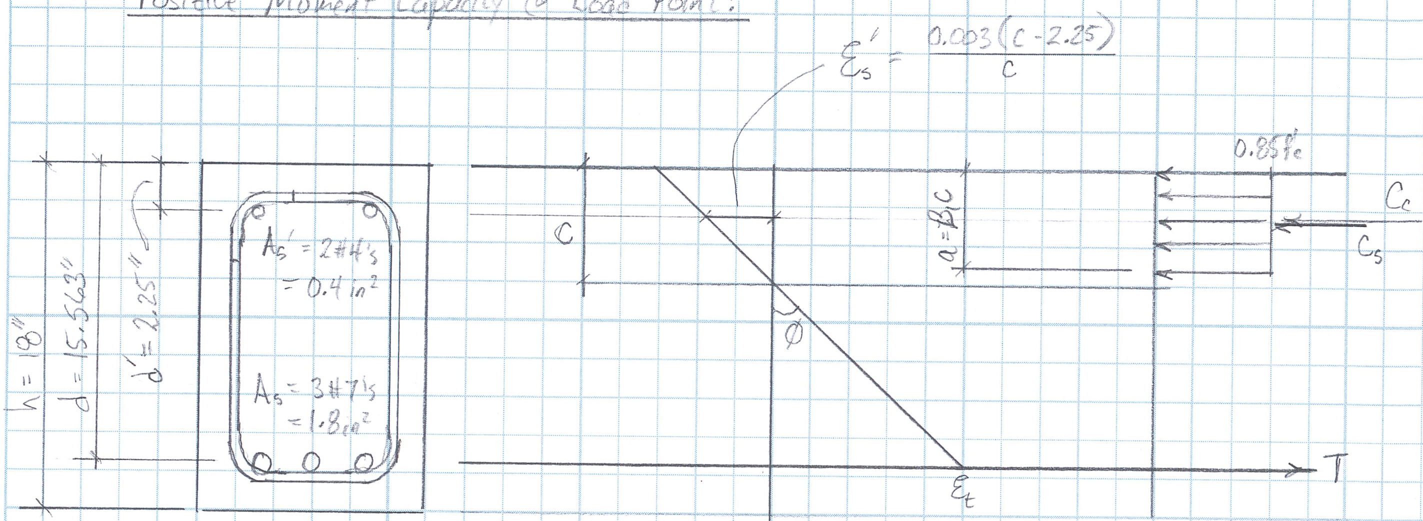
Positive Moment Capacity @ Load Point:



$f_c' = 6.5\ ksi \Rightarrow \beta_1 = 0.725$
$f_y = 69\ ksi \Rightarrow \varepsilon_y = 0.0024$

Assume A_s yields, A_s' does not

$\Sigma F_x = 0 \Rightarrow C_c + C_s = T$

$$(0.85f_c')\left[(\beta_1 c)(b) - A_s'\right] + A_s' E_s \varepsilon_s' = A_s f_y$$

$$(0.85)(6.5)\left[(0.725)(c)(12) - 0.4\right] + (0.4)(29000)\frac{(0.003)(c-2.25)}{c} = (1.8)(69)$$

$\Rightarrow$ $c = 2.546\ in$

$\rightarrow C_c = (0.85)(6.5)\left[(0.725)(2.546)(12) - 0.4\right] = 120.17^k$ (c)

$C_s = 0.4(29{,}000)\frac{(0.003)(2.546-2.25)}{2.546} = 4.05^k$ (c)

$T = (1.8)(69) =$ 124.2^k (t)

check: $0.02 \approx 0$ ✓

note: $\varepsilon_s' = 0.00035 < \varepsilon_y$ ✓

$\varepsilon_t = \frac{(0.003)(15.563-2.546)}{2.546} = 0.0153 > \varepsilon_y$ ✓ (highly ductile)

$$\Sigma M = 0 \Rightarrow M_n = C_c\left(d - \frac{a}{2}\right) + C_s(d - d')$$

$$= 120.17\left(15.563 - \frac{(0.725)(2.546)}{2}\right) + 4.05(15.563 - 2.25) = 1813\ in\text{-}k$$

$$= 151'^{k}$$

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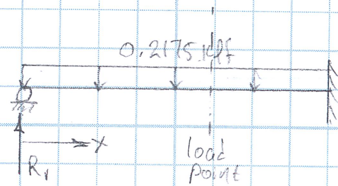
Assuming Elastic Beam and $w = \frac{(12)(18)}{144}(145\ \text{lb}/\text{ft}^3) = 217.5\ \text{lb}/\text{ft}$

For self-weight: R_1 @ roller $= 3wl/8 = 3(217.5)(24)/8 = 1.958\ k$

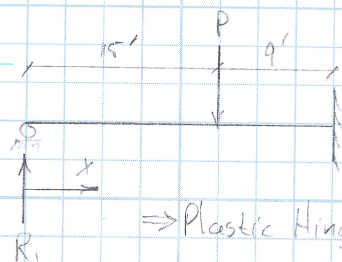


$M_{Fixity} = {}^{-}wl^2/8 = (217.5)(24)^2/8 = {}^{-}15.66^{1k}$

$M_{load\ point} = R_1 x - \frac{wx^2}{2}$ where $x = 15'$

$= 1.958(15) - \frac{(0.2175)(15)^2}{2} = 4.89^{1k}$

For point load: R_1 @ roller $= Pb^2(a+2l)/2l^3 = P(9^2)(15+2(24))/2(24)^3 = 0.1846P$



$M_{Fixity} = {}^{-}Pab(a+l)/2l^2 = {}^{-}P(15)(9)(15+24)/2(24)^2 = {}^{-}4.570P$

$M_{load\ point} = R_1 a = (0.1846P)(15) = 2.769P$

$\Rightarrow$ Plastic Hinge will form @ fixity when

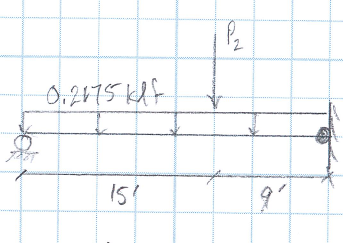
$4.57P_1 + 15.66 = 164^{1k} \Rightarrow P_1 = 32.46^k$

(note: to form initial hinge @ load point, $2.769P_1 + 4.89 = 151^{1k}$
$\Rightarrow P_1 = 52.77^k$)

when $P_1 = 32.46^k$, $M_{load\ point} = 4.89 + 2.769P = 94.77^{1k}$

$\Rightarrow$ Reserve moment capacity @ load point $= 151^{1k} - 94.77^{1k} = 56.2^{1k}$

Once Plastic Hinge forms @ load point, beam will behave like a simple span



Moment @ load point due to self weight $= wx(l-x)/2$

$= 0.2175(15)(24-15)/2 = 14.68^{1k}$

Moment @ load point due to additional load, $P_2 = P_2(ab)/l = 5.625P_2$

so $14.68 + 5.625P_2 = 56.2^{1k} \Rightarrow P_2 = 7.39^k$

$P_{total} = P_1 + P_2 = 32.46 + 7.39 = \underline{\underline{39.85^k}}$

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It should be noted that the grade 60 steel used to reinforce the beam apparently begins to strain harden almost immediately upon yielding. For strain in the range of 0.015 for our highly ductile plastic moment regions, the stress in the steel will be around 80 ksi, corresponding to a 16% increase over f_y.

Re-computing moment capacities using $f_s = 80\text{ksi}$ gives

M_u @ fixity = $188^{'k}$

M_u @ load point = $174^{'k}$

$\Rightarrow P_1 = (188 - 15.66)/4.57 = 37.7^k$

$\Rightarrow P_2 = (64.62 - 14.68)/5.625 = 8.88^k$

$\Sigma = P_{tot} = \underline{\underline{46.6^k}}$

(after accounting for strain hardening)