Thermal comfort evaluation for mechanically conditioned buildings using response surfaces in an uncertainty analysis framework

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An uncertainty analysis methodology is proposed to aid in quantifying the risks of thermal comfort under-performance posed by changes to variations in physical and operational characteristics of a building and its environment. This includes those implemented for building energy savings, peak electricity load reductions, or those due to climatic changes. Using building performance data as input, a response surface methodology is used to develop a model to predict building thermal performance for ranges of user-defined design variables. This model is verified for accuracy using in- and out-of-sample data. Uncertainty analysis is then used to estimate the probability of achieving an acceptable threshold of thermal comfort performance. A case study is presented to demonstrate the implementation and interpretation of the results of this methodology, which evaluates the effects of a 1-h demand response event on thermal comfort of a residential mechanically-conditioned building. The case study finds that a second-order response surface provides a reasonably accurate model of thermal comfort. For the studied single family home, compared to varying the air exchange rate, the indoor set-point temperature has a greater influence on achieving an acceptable level of thermal comfort.

Introduction

In many developed countries, on average, people spend 80–90% of their time indoors (U.S. EPA 1989; Leech et al. 2000) in buildings. The thermal comfort of the occupants, a measure of the satisfaction with the indoor environmental conditions, is thus of great importance, and has been linked to the health, well-being, and productivity of occupants (e.g., Schellen et al. 2010; Akimoto et al. 2010; Almeid-Silva et al. 2014). To provide a comfortable and productive environment, buildings also consume a significant amount of energy, and are one of the largest consumers of energy in the United States (U.S. Energy Information Administration [EIA] 2013). To reduce energy use and costs, and to reduce greenhouse gas emissions, buildings continue to be targeted through energy-efficient retrofits. To improve electric grid reliability peak load reduction programs also targets buildings, in particular their system operations. However, as these measures are implemented, these changes to building physical and operational characteristics also affect buildings’ thermal performance and thus can also affect occupant thermal comfort. These effects should be carefully considered.

The ability to achieve a comfortable indoor environment for occupants is influenced by many design variables, including building envelope and systems characteristic, internal loads, and external environmental conditions. Many of these are summarized in Table 1. Their values can also vary significantly between buildings. These same design variables can also influence the energy (kilowatt-hour) and instantaneous load (kilowatt) contributions of a building. As HVAC use is often a large consumer of energy in a building, particularly in the United States, its operational strategies are also a common target for energy and peak load reduction programs and strategies (Sinao 2014; Gyamfi et al. 2013; Cetin and Novoselac 2014). Other changes to building characteristics, such as reduction to the air exchange rate (air changes per hour [ACH]) through weatherization, have been targeted by large government funded programs (U.S. Department of Energy [DOE] 2015). To date, however, there have been limited studies that provide a methodology that can evaluate the tradeoffs between peak load and energy savings measures, and thermal comfort.

Mathematical models developed by Fanger (1967, 1970, 1972) provide the basis for the most widely accepted international thermal comfort standards for mechanically conditioned buildings, including ASHRAE Standard 55 (ASHRAE 2010), International Standards Organization (ISO) 7730 (ISO 2005), and EN 15251 (CEN 2006).
Table 1. Common design variables influencing thermal comfort in mechanically conditioned buildings.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Variables</th>
<th>Effects on building interior conditions when variable increased</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVAC</td>
<td>Cooling/heating set-point (°C); deadband of thermostat (°C); HVAC cooling capacity (kilowatts)</td>
<td>Increase/decrease interior temperature; increase allowable temperature variation above/below set-point; increase HVAC ability to remove heat from interior</td>
</tr>
<tr>
<td>Building envelope</td>
<td>Air exchange rate (1/h); windows/doors, walls, roof, ground U-value (W/m²-°C); window area, interior shading (%); thermal mass (W/m²-°C)</td>
<td>Increase in unconditioned outdoor air entering building interior; increase in heat transfer between interior and exterior conditions; increase and reduce, respectively, effect of solar heat gains (temperature) to interior; slow and attenuate the effect of exterior conditions on interior conditions</td>
</tr>
<tr>
<td>Internal loads</td>
<td>Large appliances (W); occupants (W); electronics (W); hot water heater (W); lighting (W)</td>
<td>Increase in internal heat (temperature) and/or moisture (humidity) gains</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>Outdoor temperature (°C); outdoor humidity (%); solar radiation (Wh/m²)</td>
<td>Increase internal heat gains (temperature); increase internal moisture gains (humidity); increase internal heat gains (temperature)</td>
</tr>
</tbody>
</table>

Conditions that are considered in defining acceptable thermal comfort of building occupants include (1) environmental factors such as: dry-bulb air temperature (°C), mean radiant temperature (°C), air speed (m/s), and humidity (%), and (2) personal factors consisting of: metabolic rate (met), and clothing insulation (clo) (ISO 2005; ASHRAE 2013). The polygons in Figure 1 represent the typical thermal comfort zones (TCZs) for cooling and heating seasons according to ASHRAE 55 (ASHRAE 2013), however changes in assumed level of clothing (clo) and metabolic rate (met) may be adjusted, resulting in a different location and size of the TCZ. This model however, evaluates thermal comfort at a single point in time, whereas evaluation of the effect of changes to a building’s characteristics on thermal comfort requires determining thermal comfort over a longer period of time. More recent methodologies for defining the level and severity of thermal comfort/discomfort over a period of time have been proposed by a number of authors. The percentage outside range (Carlucci and Pagliano 2012), hourly performance index (Hensen and Lamberts 2012), and hours of exceedance (Olesen and Brager 2004) methodologies, discussed in Standard ISO7730 (ISO 2005), count the number of hours inside and outside the TCZ, represented as a fraction of the total number of hours evaluated.

To quantify the effect of the building design variables on indoor environmental performance, including thermal comfort, building energy modeling (BEM) is often used, computer-based tools for developing a model of a building and its systems, and simulating its performance at a design location and over a defined period of time. However, carrying out a large number of BEM simulations to evaluate different scenarios is time-consuming, particularly if the goal is to take into account the uncertainties of the input variables used to evaluate building performance. Various techniques to simplify the evaluation of BEM have been proposed. Eisenhower et al (2012) developed a simplified normative model and calibrated it to BEM, based on the techniques discussed in other works (ISO 2007; CEN 2005). Reduced-order models have also been developed for the purpose of building control strategies (Goyal and Barooah 2012; Dewson et al. 1993). Artificial neural networks (ANNs) have also been used to develop models to predict building energy use and thermal comfort (Yuce et al. 2014; Chang et al. 2015; Ashtiani et al. 2014).

The response surface methodology (RSM) is another technique for the study of the relationship between a measured response and a set of design (input) variables (Box and Wilson 1951). The use of RSM has several advantages. Between the upper and lower bounds of each variable considered, RSM includes a large amount of information from a limited number of controlled experiments. It can be used in reducing the computational cost of expensive analysis methods such as finite element analysis (Guan and Melchers 2001; Reh et al. 2000). For example, Fig. 1 shows the psychometric chart displaying TCZs for cooling and heating seasons according to ASHRAE 55 (ASHRAE 2013).
A multi-step methodology is proposed to evaluate building thermal comfort; it is presented schematically in Figure 2. It is divided into five main steps: (1) design variable definition, (2) BEM, (3) response surface development, (4) uncertainty analysis, and (5) result interpretation. Each of these steps is outlined in detail next.

**Step 1: Variable definition for response surface model development**

In evaluating options for construction or operational changes of a building, different design variables are considered. These design variables are used as inputs to build and define the response surface. These design variables can include physical building characteristics such as window area and wall construction, operational characteristics such as thermostat set-points and fan schedules, or climatic characteristics, such as the location of the building and potentially even future climate change scenarios. Building, operational, and climatic characteristics that may affect thermal comfort in buildings are included in Table 1. To develop a response surface for use in this study, the design variable vector, \( \mathbf{X} = \{ X_1, X_2, \ldots, X_n \} \) of size \( n \) must be chosen. The greater the number of variables \( n \) there are, the greater will be the computational effort required to evaluate all possible combinations of the design variable values employed to construct the response function. A larger number of design variables allows definition of a more generalized response surface to describe the response of the building.

Each design variable \( X_i \) is defined by its mean value \( \mu_i \), a standard deviation \( \sigma_i \), and a probabilistic distribution function. In order to determine the probability distribution of each variable, appropriate distributions from the literature and related studies may be used. An Anderson-Darling test may also be performed on a dataset to determine the best distribution fit. Upper and lower bounds are chosen for each design variable. Following Wong (1985) and Faravelli (1989), \( X_{i,\text{high}} \) and \( X_{i,\text{low}} \) are selected as upper and lower bounds (\( r_i \) standard deviations above and below the mean, respectively) for design variable \( X_i \) to be evaluated in the RSM (Equations 1a and 1b):

\[
X_{i,\text{high}} = \mu_i + r_i \sigma_i, \quad \text{(1a)}
\]

\[
X_{i,\text{low}} = \mu_i - r_i \sigma_i, \quad \text{(1b)}
\]

Caution should be exercised if the RSM is used in uncertainty analysis, to evaluate the system response outside of posed methodology may be implemented for a mechanically conditioned building as a tool to evaluate thermal comfort for any user-defined range of a set of design values. This methodology may be applied both in the design phase of a building when evaluating energy savings strategies versus the risk of discomfort, and for existing buildings in which operational or physical changes to the buildings are being evaluated for use in building energy use or peak electricity use reductions.
the upper and lower bounds of each design variable, as doing so may provide an inaccurate assessment of the response function. Values for $\mu$, $\sigma$, and the probabilistic distribution function for each design variable $X_i$ may be selected based on documented studies of building characteristics as well as operational and climatic considerations (Persily 1998, 1999; Air Tightness Testing & Measurement Association [ATTMA] 2010; Chartered Institution of Buildings Services Engineers [CIBSE] 2000; Offermann 2009; ASHRAE 2004; Persily et al. 2010; Parker et al. 1990; Roberts and Lay 2013). They may also be chosen following a data collection effort or by using engineering judgment. For example, if a set of existing homes is being considered for energy-efficient retrofit strategies and one of the design variables is the window area (measured in square meters), the window area may be measured for each of the buildings considered and a mean, standard deviation, and distribution function may be derived directly from the data. The choice of $\sigma$ in Equations 1a and 1b determines the upper and lower bounds of the range of values of each variable for which the RSM may be assumed to be valid. As an example of the use of engineering judgment, if indoor temperature is a variable, previous studies have reported average indoor temperatures, such as those summarized by Roberts and Lay (2013; Hammersley et al. 1964). These could be related to the study for which the response surface model is being developed, to define design variable ranges, statistics, and distributions.

If a year-long period is used, since there are different TCZ criteria for heating and cooling seasons, a reasonable division of the year into heating and cooling seasons may be made consistently for all the BEM simulations considered. Portions of the year representing a cooling season or a heating season may each be evaluated, provided the same period of time of the year is considered for each season in all the BEM simulations carried out.

Heating and cooling only occur during certain months of the year. These seasons can be determined using monthly average temperatures (MATs) and typical meteorological year (TMY3) data (Wilcox and Marion 2008), or the 99% annual winter and summer and design temperatures as defined by ASHRAE (ASHRAE 2009). All months where the MAT or 99% design temperature is less than 18.9°C are defined as the heating season, and all months where the MAT is greater than 18.9°C are defined as the cooling season. Additional information on this methodology is included in the Building America House Simulation Protocols (Wilson et al. 2014) used for building energy simulation.

### Step 2: BEM simulations

In the present study, to establish the desired response surface, input data on the thermal comfort performance of the subject building are needed. Such data include consistent time-interval data of, typically hourly, the indoor operative temperature (°C), or both the dry-bulb temperature (°C) and the mean radiative temperature (°C). Data indicating relative humidity (%) or humidity ratio (grams/kilograms) of the indoor air could also be included. The required data may be obtained using results from BEM or from field-collected building performance studies. The use of building energy simulation results is the more cost-effective methodology as field testing is expensive and takes far more time and effort than simulations. In the present study, BEM is used to produce the indoor operative temperature and humidity ratio data; it is assumed that air speed criteria (ASHRAE 2010; Gyanfie et al. 2013) for thermal comfort are met in the analyses.

In addition to a consistent time interval for measurements or simulated values, both the design period of evaluation over the calendar year and the design time of day must be chosen. In reporting the results of the methodology employed in this study, all the assumptions, including those discussed here, should be explicitly stated so that the results are not misinterpreted, as discussed by Carlucci and Pagliano (2012).

A design period is defined by a start day $d_{start}$ and an end day $d_{end}$. Thus, the day of simulation $d$ is such that $d_{start} \leq d \leq d_{end}$, and the total number of days evaluated is $d_{tot}$. One year ($d_{start} = 1; d_{end} = 365, d_{tot} = 365$) may be used to capture the behavior of the building accounting for all seasons of the year, a single year is a typical period of time used in BEM studies.

A nonlinear response surface is constructed using 3rd BEM simulations. This includes a simulation at each combination of the $n$ design variables ($X_i; i = 1$ to $n$) at three design points, $X_{i,high}, X_{i,low}$, and $\mu_i$. Once the BEM simulation results are generated, the percent of time inside and outside the TCZ must be computed from each simulation. With a defined TCZ, such as in Figure 1, each simulated time interval data point for the selected design time period is plotted on the psychometric chart to determine its location relative to the TCZ for that season. The percent of simulated data points that lie outside the TCZ, $S_{k, data}$, where $k = 1$ to $3^v$, is computed using Equations 2a and 2b, where a value of 1 for each time interval data point indicates that the simulated point is outside the TCZ while a value of 0 indicates it is inside the TCZ:

$$S_{k, data} = \left( \frac{\sum_{i=1}^{n} (h_{i} - h_{i, start} - c_{i,k})}{d_{start}w_{tot}} \right), \quad (2a)$$

$$c_{d,h} = \begin{cases} 1 & \text{outsideTCZ} \\ 0 & \text{insideTCZ} \end{cases} \quad (2b)$$

If a large number of design variables are being evaluated, the number of simulations needed (3$^v$) for the full factorial design may become computationally expensive. In this case,
methodologies such as the fractional factorial design (Gunst and Mason 2009), Box–Behnken design (Box and Behnken 1960), or D-optimal design (Silvey 1980) may be used to reduce the number of BEM simulations needed. These designs are desirable when the extreme points are expensive or impossible to test, or when the full factorial design requires too many runs for the amount of resources or time available.

**Step 3: Response surface development**

The third step in the methodology adopted involves development of the response surface. RSM generally assumes the use of a low-order polynomial response function $S$, which is an approximation of the measured response of the system under consideration. This response function may be defined using a set of linear and/or nonlinear terms made up of $n$ design variables $X = \{X_1, X_2, \ldots, X_n\}$ and including a set of model coefficients $b_i$ ($i = 1$ to $n$) for linear variation and $b_ij$ ($i, j = 1$ to $n$) for quadratic variation, along with a random experimental error term $\varepsilon$. Simpler response functions are often of first-order (Equation 3a) or second-order (Equation 3b) forms (Khuri and Mukhopadhyay 2010):

$$S(X) = b_0 + \sum_{i=1}^{n} b_i X_i + \varepsilon.$$

$$S(X) = b_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} X_i X_j + \varepsilon.$$

Additional information on response surface creation is discussed in previous works (Meyers et al. 2009; Khuri and Mukhopadhyay 2010; Meyers et al. 1989). Least-squares regression is used with the selected design variables (Step 1) and the BEM simulations (Step 2) to build the response surface. To evaluate the goodness of fit of the regression model to the data the $R^2$ (coefficient of determination) value is used. A good fit of the response surface to the data is indicated by an $R^2$ value close to unity. Evaluation of goodness of fit should be conducted on both in-sample data used to develop the response surface as well as on out-of-sample data that were not used to develop the response surface, but are within the range of the upper and lower bounds of the design variables considered in the study.

**Step 4: Uncertainty analysis**

The response surface model developed following BEM simulations is an approximate representation of a real-world based situation based on assumptions and approximations. To address uncertainty in the underlying design variables, $X$, a limit state function (Equation 4), $g(X; T_{acc})$, is used to quantify the probability of exceeding the acceptable percent of time $T_{acc}$ outside the TCZ. Note that $S(X)$ represents the predicted number of hours outside the TCZ based on the response surface defined by Equation 3b, which is built using the design variables. One assumes that all the design variables, $X_i$ ($i = 1$ to $n$), can be treated as independent random variables:

$$g(X; T_{acc}) = T_{acc} - S(X). \quad (3)$$

To achieve compliance with generally accepted standards (ASHRAE 2010), as a part of the design of a building, the maximum allowable percent of time outside the TCZ must be stated. Monte Carlo simulations (Hammersley et al. 1964) can be used with assumed distributions for all the design variables ($X$) and with the developed response surface, $S(X)$, and the specified value of $T_{acc}$. A “failure” in a single Monte Carlo simulation is defined to have occurred when $S(X)$ exceeds $T_{acc}$ or, effectively, when $g(X; T_{acc})$ is less than zero. Crude Monte Carlo (CMC) simulation, i.e., Monte Carlo simulation without any additional variance-reduction refinement, is used in this manner to estimate the failure probability $P_f$, which is the probability of exceeding the allowing percent of time outside the TCZ. An alternative procedure referred to as the first-order reliability method (FORM) can also be used to estimate $P_f$; in this procedure, the notion of a limit state function (here, $g(X; T_{acc})$) is used along with the design variable vector definition to estimate $P_f$ more efficiently than with CMC simulations. The accuracy in $P_f$ estimates based on CMC simulations increases with the number of simulations.

**Step 5: Result interpretation**

The methodology presented in the preceding four steps provides a means of evaluating a range of physical, operational, and environmental characteristics of a building as well as its proposed environment from the point of view of thermal comfort. The results of Steps 1 to 3 provide the response surface function (a polynomial built using BEM simulations) that defines the percentage of time outside the TCZ based on $n$ design variables. This response surface may then be used to evaluate the thermal comfort response of the considered building due to other values of the design variables that lie between the upper and lower bounds used to build the response surface. Multiple sets of CMC simulations allow the systematic study of the design variables and their importance. An example of the overall analysis and interpretation of the results is provided in the illustrative case study presented next.

**Case study**

There are many different applications of the proposed methodology that can benefit from understanding building occupants’ risk of exceeding a specified number of hours outside the TCZ. A case study is presented to describe the effect on thermal comfort of executing a single hour of air conditioner-based demand response during the summer months for homes in Austin, TX. This involves turning off the air conditioner of homes during times when there is greatest load on the electric grid. According to historical data from Electric Reliability Council of Texas (ERCOT), this often occurs at around 5:00 pm during the summer (ERCOT 2013). In this case study, one assumes that the air conditioner is shut off for 1 h from 5:00
pm to 6:00 pm. The characteristic home used in this study is a single-family detached home (114-m², three-bedroom, two-bathroom home) located in Austin, TX. The studied home includes a single-stage residential HVAC system with an outdoor compressor and condenser unit and indoor air handling unit. Cooling and heating are electric-based from a heat pump. The air distribution system and duct system are located in the attic. The size of the HVAC system was fixed based on Manual J (Rutkowski 2011) sizing calculations for the studied climate zone assuming a constant cooling set-point and the mean values of the properties of the studied variables listed in Table 2. Internal loads are based on typical occupancy and internal load schedules for residential buildings from the Building America Energy Simulation Protocol (Hendron and Engebrecth 2010). These include major household appliances, including a refrigerator, clothes washer and dryer and dish-washer, as well as other miscellaneous loads. The building envelope properties are based on the building code requirements of the International Energy Conservation Code (2009) for this climate zone. This code specifies minimum thermal properties of the building envelope for newly constructed buildings. R-values of the ceiling, walls, and windows are R-30, R-13, and R-2, respectively, with a window solar heat gain coefficient of 0.30. The window area was assumed to be 15% of the total exterior wall surface area.

Austin, TX is located in a hot-humid climate zone, ASHRAE climate zone 3a (ASHRAE 2013). To simulate the outdoor conditions of this climate zone a TMY3 (Wilcox and Marion 2008) weather file was used, which is developed from weather data from Class I weather station data. Based on this data, during the summer time period of study (May 1–September 30), the dry-bulb temperature ranges from 6.1°C to 38.9°C with an average and median temperature of approximately 26°C. The cooling degree days (CDD) total 2537 using a reference temperature of 10°C. The relative humidity during this period ranges from 22% to 100%, with an average and median of 71%–72%. The corresponding dew point temperatures were average and median of 20°C–21°C with a range of 5°C to 26°C.

Two design variables \((n = 2)\) are chosen as a case study; these include the average indoor cooling set-point temperature \((°C)\), assuming a single zone model, and the whole-home air exchange rate \((\text{ACH, hr}^{-1})\). These design variables directly affect the two main variables that determine thermal comfort: temperature and relative humidity. The set-point temperature directly affects the indoor temperature. The level of ventilation indirectly affects relative humidity. During the summer period of study when changing the HVAC system use, in this case study due to a demand response event, these are particularly important. During this period, infiltration brings moisture into a house in a humid climate, thus the indoor relative humidity can increase due to increased ventilation. The air-conditioning unit dehumidifies a home, however since the thermostat is driven by the total load, and not just the ventilation-caused loads, an increase in ventilation also can mean higher relative humidity. In addition, these design variables are easily adjusted by the building owner or occupant of the building. The cooling set-point temperature may be changed through adjusting the thermostat settings and the air exchange rate may be adjusted through weatherization techniques. This methodology can be expanded to include additional design variables, included those listed in Table 1. However two are chosen to provide a proof of concept of the proposed methodology. Set-point temperature determines the target indoor temperature of the building under consideration and directly affects the indoor thermal comfort. The upper and lower bounds of the set-point temperatures were to be within the upper and lower limits of the TCZ. The air exchange rate \((\text{ACH})\) affects the amount of unconditioned exterior air that is exchanged with conditioned interior air. A higher \(\text{ACH}\) means that when there is a difference between the outdoor and indoor conditions, the indoor conditions follow outdoor conditions closely, such that the HVAC system must work longer to meet the desired indoor conditions. \(\text{ACH}\) can vary significantly across residential buildings, with newer homes with tighter building construction having a lower \(\text{ACH}\), and older, leakier homes having a higher \(\text{ACH}\). The upper and lower bounds were chosen to cover a range of values common in newer buildings, or older buildings in which weatherization measures have been installed. Details related to these design variables are presented in Table 2. The mean and standard deviation values for each of the design variables were determined using a building energy use dataset collected for single family homes in the Austin, TX area (Pecan Street Research Institute 2011). An Anderson–Darling test was performed to determine the best distribution fit to the data for each of the design variables based on the referenced dataset. Only the summer, i.e., the cooling season, is evaluated such that \(d_{\text{start}} = 121, d_{\text{end}} = 273, \text{ and } d_{\text{tot}} = 153\). All data are in hourly intervals and all hours of the day are included in the analysis such that \(h_{d,\text{start}} = 1, h_{d,\text{end}} = 24, \text{ and } h_{\text{tot}} = 24d_{\text{tot}} = 3,672 \text{ h}.\) Since there are two design variables, \(x^2\)
or 9 simulations are carried out to construct the response surface.

BEM simulations were run using the EnergyPlus software (U.S. DOE 2007) and available weather data for Austin, TX (Wilcox and Marion 2008). The output data of the BEM included the hourly operative temperature and humidity ratio. A matrix laboratory (MATLAB) code was created and run using the output data of the BEM, to determine the number of hours outside of the TCZ. The TCZ assumed clothing insulation of 0.5–1 clo and a metabolic rate of 1.1 met. The resulting number of hours outside the TCZ for each BEM simulation is shown in Table 3. Since both the operative temperature and the humidity ratio influence this value, their relative contributions are also included in Table 3. Plots of the extreme cases of 20 h (0.5%) and 695 h (18.9%), Simulation numbers 5 and 9 in Table 3, are shown in Figures 3a and 3b.

Least-squares regression is carried out to develop the non-linear response surface function, \( S(X) \) (Equation 5). The estimated \( R^2 \) is 0.982. A comparison of the predicted (RSM) and simulated data indicating the time outside the TCZ is shown in Figure 4a. To verify the accuracy of the RSM, a set of eight randomly selected values for \( x_1 \) and \( x_2 \) are chosen within the upper and lower bounds from Table 2. BEM simulation was conducted using these values and evaluated against the predicted values from the RSM; these are shown in Figure 4b with an \( R^2 \) of 0.965:

\[
S(X) = 4.73 - 0.41x_1 - 0.80x_2 + 0.032x_1x_2 + 0.0089x_1^2 + 0.176x_2^2 + \varepsilon
\]  

(4)

For this case study, three values of \( T_{acc} \) are considered corresponding to 5%, 7%, and 10% of the time it is acceptable to be outside the TCZ. These values were chosen to explore the response of a range of acceptable levels of thermal comfort, and are based on recommendations in standard EN 15251 (CEN 2006), a similar standard to ASHRAE 55 (ASHRAE 2013), which discusses thermal comfort criteria. EN 15251 suggests that no more than 3%–5% of the occupied hours of a given period of study should be outside the limits of the specified TCZ. This study thus explores a range of values, from this recommended percentage to two times this percentage (5%–10%). The limit state function \( g(X; T_{acc}) \) is evaluated for each of these values of \( T_{acc} \) to estimate the probability that each of these design allowable percentages of time outside the TCZ is exceeded. A total of 10,000 CMC simulations are run using the design variable characteristics given in Table 2. Since the RSM was developed using energy simulations out to \( \pm 3\sigma \) for each variable, the polynomial function is valid for the values within this range of each design variable.

Figures 5a to 5e summarize the results of this simulation. Figures 5a, 5c, and 5e show the estimated probability of exceeding the maximum allowed percent of time \( T_{acc} \) outside the TCZ as a function of air exchange rate for fixed set-point temperatures for \( T_{acc} \) equal to 5%, 7%, and 10%, respectively. Similarly, Figures 5b, 5d, and 5f show estimated of the probability of exceeding the maximum allowed percent of time \( T_{acc} \) outside the TCZ as a function of set-point temperature for fixed air exchange rates for \( T_{acc} \) equal to 5%, 7%, and 10%, respectively. By choosing a single fixed set-point temperature (as in Figures 5a, 5c, and 5e), or a single fixed air exchange rate (as in Figures 5b, 5d, and 5f) trends in how sensitive the probability of exceeding \( T_{acc} \) is to the other varying parameter are evident.

In Figure 6, the variation in probability of exceeding the maximum allowed percent of time \( T_{acc} \) outside the thermal comfort as a function of air exchange rate is studied for a single indoor set-point temperature fixed at nearly its mean value (24°C) and for three different values of \( T_{acc} \) (5%, 7%, and 10%). Similarly, in Figure 6b, the variation in probability of exceeding the maximum allowed percent of time \( T_{acc} \) outside the thermal comfort as a function of set-point temperature is studied for a fixed single air exchange rate fixed at its mean value (0.26 ACH (h⁻¹)) and for three different values of \( T_{acc} \) (5%, 7%, and 10%).

**Discussion**

The value of the use of the response surface and uncertainty analysis is that by using the response surface developed, a con-
tinuous range of values for any design variable may be evaluated easily without carrying out any additional BEM simulations beyond what were run to construct the response surface. The results of this case study show that with increasing values of $T_{acc}$, the probability of exceeding this allowed percentage of time outside the TCZ decreases; this is not unexpected. If occupants are more tolerant of a greater amount of time outside the TCZ, the risk of exceeding that threshold will naturally be reduced. Comparing the influence of the indoor set-point temperature (°C) and that of the air exchange rate (ACH, h$^{-1}$), one finds that a change in set-point temperature has a greater effect on the probability of exceeding $T_{acc}$ than does the air exchange rate. Comparing a home or set of homes with a lower average indoor cooling set-point temperature (22.5°C) to a higher one (26.5°C), the probability of exceeding any selected $T_{acc}$ value increases by 70% to 100% in all cases (Figures 5b, 5d, and 5f). On the other hand, homes with a lower value for air exchange rate (0.15 ACH (h$^{-1}$)), compared to those with a higher average value (0.4 ACH (h$^{-1}$)) leads to a change in the probability of exceeding $T_{acc}$ by between 3 and 20% (Figures 5a, 5c, and 5e). The influence on changes to the probability of exceeding $T_{acc}$ for the range of values of

$T_{acc}$ studied (5%–10%) is greatest at high air exchange rates (above 0.35 ACH (h$^{-1}$)) and at higher set-point temperatures (25°C–26°C) (Figures 6a and 6b).

For the single family home evaluated in this case study, the results of the response surface model development and the uncertainty analysis provide combinations of the design variables that will meet specified thermal comfort requirements of the occupants. The results of the uncertainty analysis quantify the likelihood that these specified comfort requirements are met. For example, if an occupant of the considered building wants to have 90% confidence (i.e., $P_f = 10\%$) that he/she will be outside the TCZ only 5% of the time, the indoor set-point temperature can be set as high as 24.5°C as long as the air exchange rate is extremely low. At a higher air exchange rate (around 0.5 ACH), typical of an older home, the set-point temperature must be set to 23.3°C, more than a degree lower. The graph presented in Figure 7 shows upper bounds of acceptable parameters for the case study home covering various situations where the 90% confidence and 95% confidence curves correspond to $P_f$ values of 10% and 5% of the time outside the TCZ (when $T_{acc}$ is set at two different values).

![Fig. 3. BEM hourly data results for specific simulations. a. With the largest number of hours ($x_1 = 26.7°C, x_2 = 0.47$ 1/h) outside the TCZ (shown in blue). b. With the smallest number hours ($x_1 = 23.9°C, x_2 = 0.05$ 1/h) outside the TCZ (shown in blue).](image)

![Fig. 4. Comparison of the percent of time outside the TCZ. a. Based on the in-sample BEM simulations and the response surface prediction. b. Based on the out-of-sample BEM simulations and the response surface prediction.](image)
Note that the results in Figure 7 show how the uncertainty analysis with Monte Carlo simulation can be used to address specific “design” requirements where one is interested in combinations of the design variables (set-point temperature and air exchange rate, here) to meet desired TCZ levels with a target level of confidence. An alternative and more efficient approach to Monte Carlo simulations is to use “inverse reliability” approaches where the target level of confidence is the starting point and candidate values of the design variables are directly derived using information on the underlying random variables (Winterstein et al. 1993; Saranyasontorn and Manuel 2004a, 2004b, 2006).

Limitations

There are several limitations of the present study. The main one is related to the sources of possible error in the results that arise from each of the five steps in the methodology. The results are limited by the uncertainty in the statistics and probability distributions of the design variables. In some cases, required
statistics and distributions may not be readily available. One solution then is to use expert engineering judgment in selecting suitable statistics (de Wit and Augenbroe 2002). BEM, as it employed in this study, relies on many simplifying assumptions; also, not all the various design variables are considered in the RSM. Assumptions both in the BEM and RSM need to be recognized and should provide context and bounds for situations the end results can be applied to, when the methodology presented here is applied.

In the development of the response surface for this study, three values for each design variable were considered in developing the response surface; thus, $3^n$ BEM simulations were used. Additional points beyond the upper and lower bounds and the mean value for each design variable would improve the accuracy of the response surface. This would also increase the computational time needed to develop the response surface from the BEM simulations. When compared to both in-sample and out-of-sample BEM simulations, the response surface provides a good fit with 1.8% and 3.5% errors, respectively. However, particularly in cases where the amount of time outside the TCZ is low, the response surface can predict values even below zero. However, these cases near 0% of time outside the TCZ are less likely to represent situations in which the occupant thermal comfort is significantly affected. CMC simulation studies also have limitations. CMC probability estimates have uncertainty associated with them; this is only reduced when a large number of simulations are carried out.

The methodology proposed here can benefit from additional analysis and development beyond that dealt with in the limited scope of this study. The case study considered a single-zone building energy model evaluation and used one indoor set-point temperature. If a larger and more complex building is evaluated, an average or weighted average of multiple indoor parameters at different locations of the building may need to be considered. The proposed methodology may also be applied to other building performance characteristics that are affected by changes to the building’s physical and operational properties as well as to other environmental parameters. In the present study, the authors only took into account the amount of time outside the TCZ; in general, it may be of interest to consider the severity of the indoor environmental conditions, relative to ideal indoor conditions. For instance, instead of weighting all the data points with temperatures between 28°C and 32°C equally as not meeting thermal comfort requirements, one could assign a greater weight to higher temperatures, as they likely bring more severe thermal discomfort. In addition, while the temperature and humidity conditions within the TCZ are defined as being at acceptable levels to occupants, not all occupants will be satisfied to the same degree. All conditions within the TCZ may represent different levels of comfort rather than a uniform comfort level especially far from the edges of the thermal comfort. These are subjects of ongoing and future work.

**Fig. 6.** Probability of exceeding the maximum allowed percent of time $T_{acc}$ outside the thermal comfort for situations. a. Where the indoor set-point temperature is fixed at its mean value ($x_1 = 23.9^\circ$C). b. Where the air exchange rate is fixed at its mean value ($x_2 = 0.26$ ACH (1/h)).

**Fig. 7.** Acceptable combinations of indoor set-point temperature ($^\circ$C) and air exchange rate (ACH, 1/h) for specified values of $T_{acc}$ that guarantee desired levels of confidence (1–$P_f$) in meeting thermal comfort requirements of occupants.

**Conclusions and applications**

This research study proposes a five-step methodology to assess the thermal comfort of a building based on building energy simulations over ranges of selected multiple design variables. Using the results from these simulations, a response surface
describing the percent of time outside the building occupants’ TCZ is constructed. This response surface provides an empirically derived polynomial function that relates building thermal comfort performance to the design variables. Uncertainty analysis is then carried out by defining a limit state function that incorporates the response surface and a user-defined limit or threshold for acceptable thermal comfort conditions. The results provide bounds on design variable values, such as the air exchange rate and set-point temperature, that will meet the design needs with a specified level of confidence (e.g., one can arrive at combinations of design values that can guarantee with 95% probability that the percent of time spent outside the TCZ will not exceed some specified value, say 10%). This methodology is applied to a case study to demonstrate the overall procedure and result interpretation.

There are many potential applications of the proposed methodology beyond the case study. It is the authors’ belief that the use of uncertainty analysis and response surface development is the first of its kind that has been applied to such studies related to building energy and occupant comfort. Today, BEM is used mostly for the development of buildings, such as to achieve desired green building energy ratings; this study suggests that the same building energy model may also be used to conduct a thermal comfort analysis to assess the effects of proposed design strategies on thermal comfort. This may prove valuable in balancing the risk of discomfort against energy savings. It is easy to envision an extension of the methodology presented here to consider complex multivariable comfort “zones” beyond the single one used here. Finally, for utility companies that target customers for demand response, tiered electricity rate structures and other load reduction and load shedding techniques, the results of the proposed methodology may prove valuable in identifying the best customers to target and in making recommendations to residential customers to aid in load shedding while assuming low risks of thermal discomfort.

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